Misspecification, Sparsity, and Superpopulation Inference with Large-Scale Social Network Data

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Overview: Foundations of Applied Statistics

Research agenda: A theory of Applied Statistics.

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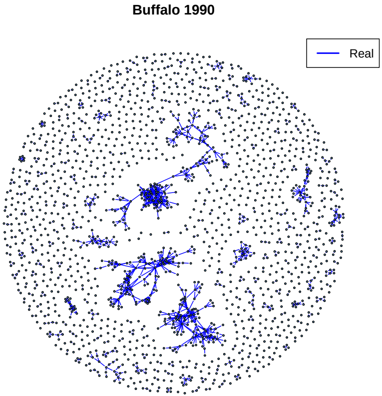
New desiderata that we can work into modeling decisions.

Generative Network Models



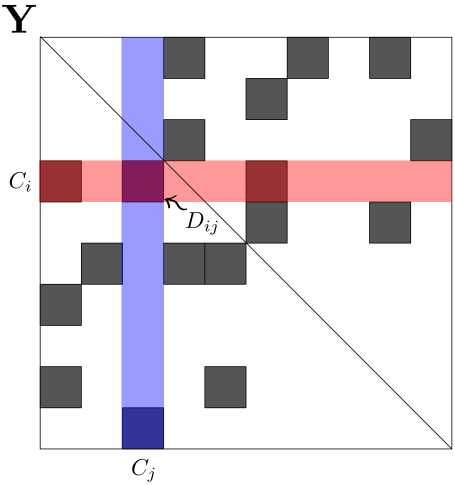
Obtain a set V of n actors.

Explain or predict pairwise outcomes Y_V, potentially using pairwise covariates X_V.

X_V may be observed or latent.

Running example: inventor collaboration network.

Data Representation



Y_V entries in **arbitrary sample space** $\mathcal{Y}$.

Covariates X_V combine observed, latent attributes,

$$X_V^{ij} = f(C_V^i, C_V^j, D_V^{ij}).$$

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Can we borrow strength between different actor-sets to obtain better resolution on the behavior of both? (e.g., regional random effects)

Attempt 1: Network Regression

Cox PH regression. (Perry and Wolfe, 2013)
Inventor coauthorships in Michigan's motor industry 1982-1988.

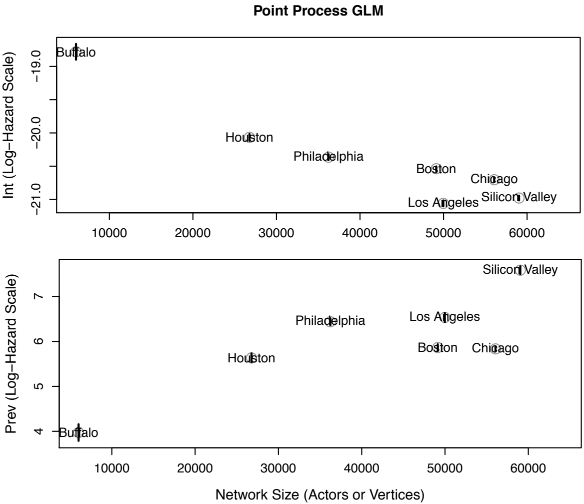
Covariates (Coefs are **log-ratios**):

- `post85`: After 1985.
- `asgnum`: Work for same firm.
- `prev`: Have worked together before.

	lower	est	upper
post85	15.49	15.84	16.20
asgnum	4.65	4.83	5.02
pre	11.36	11.73	12.10
post85:asgnum	-4.77	-4.40	-4.03
post85:prev	-14.57	-14.00	-13.44
asgnum:prev	-5.56	-5.16	-4.76
post85:asgnum:prev	3.91	4.52	5.13

Attempt 2: Regional Comparison Regression

Point process regression. Same time window, different regions.



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Inference Paradigms

What sort of question are you asking?

"Single-Sample"

For a **fixed set** of actors

- Project forward in time.
- Impute unmeasured links.

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- Compare network samples.
- Predict or pool information across networks.
- Scale local intuition to global network.

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Replications restricted to V.

Replications for any $V \subset \mathbb{V}$.

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Theory so far covers single-sample inference, giving little guidance for superpopulation questions .

Roadmap

The Question: Formalism for defining a superpopulation question.

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The Method: Criterion for usefulness of a misspecified model's MLE.

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The Result: Sparsity misspecification makes misspecified MLE's non-useful for superpopulation inference.

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Definition 1 (Random Graph Process).

A *random graph process* $Y_{\mathbb{V}}$ is a stochastic process indexed by a countably infinite vertex set $\mathbb{V}$ whose finite-dimensional distribution for any finite subset $V \subset \mathbb{V}$ defines an interaction graph Y_V with vertex set V. Denote the law of $Y_{\mathbb{V}}$ as $\mathbb{P}_{\mathbb{V}}$ and the law of a finite-dimensional projection Y_V as $\mathbb{P}_V$.

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Statistical interpretation: Observed samples are finite subgraphs of population graph. Population graph is of scientific interest.

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Example: Bickel and Chen 2009

(Y_{V_n}) a sequence of random graphs of Aldous-Hoover form where

$$\mathbb{P}(Y_{V_n}^{ij} \neq 0) = \rho_n W(C_{V_n}^i, C_{V_n}^j)$$

where $|V_n| = n$ and $\rho_n \to 0$.

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Example:
(Y_{V_n}) a sequence of random graphs derived from $Y_{\mathbb{V}}$. For each V_n, $\mathbb{P}_{V_n}$ obtained from $\mathbb{P}_{\mathbb{V}}$ by projection.

The Method: Parametric MLE

Operational procedure

Let $\mathbb{P}_{0,\mathbb{V}}$ be the law of the true population process; $\mathbb{P}_{0,V}$ be the distribution of the sample Y_V.

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Likelihood inference procedure

1. Propose a model family $\mathcal{P}_{\Theta,\mathbb{V}}$ of models $\mathbb{P}_{\theta,\mathbb{V}}$.
2. $\mathcal{P}_{\Theta,\mathbb{V}}$ implies a likelihood $\mathbb{P}_{\theta,V}$ on the sampled index set V for each $\theta \in \Theta$. Compute

$$\hat{\theta}_V = \arg\max_{\Theta} \log \mathbb{P}_{\theta,V}(Y_V). \tag{1}$$

3. Interpret $\hat{\theta}_V$ as a superpopulation parameter estimate.

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Note: Step 3 is the only difference between single-sample and superpopulation.

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Model-Building Tradeoffs

No parsimonious model can fully represent complex network structure.

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Choose one:

- **Local structure.** Homophily, Heterophily, Transitivity, etc.
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Local approaches are popular in Statistics/ML.

Models and Misspecification

Common local approaches

Conditionally independent dyads (CID, e.g., regression):

$$P(Y \mid X) = \prod_{i<j<n} P(Y_V^{ij} \mid X_V^{ij}).$$

Infinitely exchangeable dyads (Aldous-Hoover):

$$P(Y \mid X) = \int_{\mathcal{C}} \prod_{i<j<n} P(Y_V^{ij} \mid W_V^{ij}(C_V^i, C_V^j)) dF(C).$$

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Fact: Do not capture "sparsity" property of real social networks.

Misspecified Models and Science

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"Similar" inputs Y_V yield "similar" estimates $\hat{\theta}_V$.

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Huber 1967 showed MLE is large-sample consistent for a pseudo-true parameter (naming due to Sawa 1978), satisfying

$$\bar{\theta}_V = \arg\max_{\theta \in \Theta} \mathbb{E}_{\mathbb{P}_0}[\log \mathbb{P}_{\theta,V}(Y_V)] \tag{2}$$

"Similar output" defined by concentration of $\hat{\theta}_V$ in large samples.

Misspecified Models and Science

Stability: Superpopulation case

"Similar input" means any sample Y_V drawn from the same superpopulation $\mathbb{P}_{0,\mathbb{V}}$.

Intuitively, outputs $\hat{\theta}_V$ are similar if they **effectively estimate** the same thing.

What does the MLE $\hat{\theta}_V$ effectively estimate when the model is misspecified?

The Effective Estimand of the MLE

Define the effective estimand of the MLE as the finite-sample pseudo-true parameter.

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Justifications:

- Finite-sample concentration (e.g., Spokoiny 2012).
- Fisher-consistency inversion.
- Estimating equation unbiased.
- KL projection functional plug-in.

Superpopulation Stability Criterion

Criterion 1.

A procedure is superpopuatlion stable for making inferences about a superopulation process $\mathbb{P}_{0,\mathbb{V}}$ only if, for any finite sample Y_V generated according to $\mathbb{P}_{0,V}$, the effective estimand $\bar{\theta}_V$ of the estimator $\hat{\theta}_V$ is invariant to the indexing set V.

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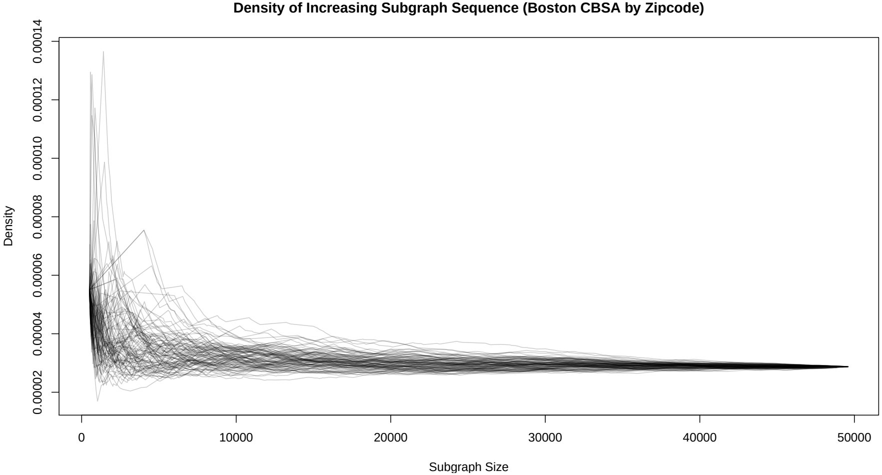
Test the criterion with top-down specification of superpopulation properties, e.g., sparsity.

The Wrinkle: Sparsity

Illustration



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Definition 1 (Sparse Graph Process).

Let $Y_{\mathbb{V}}$ be a random graph process on $\mathbb{V}$. $Y_{\mathbb{V}}$ is *sparse* if and only if for any $\epsilon > 0$ there exists an n such that for any subset of vertices $V \in \mathbb{V}$ with $|V| > n$ the corresponding finite dimensional random graph Y_V has the property $\mathbb{E}(D(Y_V)) < \epsilon$.

The Wrinkle: Sparsity

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Differences vs single-sample sparsity.

Single-sample

- Defined in terms of random graph sequences.
- Often defined in explicitly non-Kolmogorov-consistent terms.
- Analogy for single sample with very few observed interactions.

Superpopulation

- Property of a random graph process, not a random graph.
- Defines an assumption about the system, not a theoretical object.

Sparsity Misspecification

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A model family $\mathcal{P}_{\theta,\mathbb{V}}$ is **sparsity misspecfied** iff for every $\theta \in \Theta$ and every increasing sequence of vertex sets (V_n),

$$\frac{\mathbb{E}_\theta(D(Y_{V_n}))}{\mathbb{E}_0(D(Y_{V_n}))} \to 0 \text{ or } \infty.$$

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For example,

- For CID (under regularity) and exchangeable models, population extension is **dense** or **empty** (e.g., Orbanz and Roy, 2013).
- For process models, most lock in a given form for $\epsilon(n)$ (e.g., power law for preferential attachment).

Main Result: Moving Target

Assumptions

Let (V_n) be an increasing sequence of vertex sets from $\mathbb{V}$. Then assume

(A1) **Non-emptiness.** For some finite n, $\mathbb{E}_0(D(Y_{V_n})) > 0$.

(A2) **Sparsity misspecification.** The inferential family $\mathcal{P}_{\Theta,\mathbb{V}}$ is sparsity misspecified for the true population process $\mathbb{P}_{0,\mathbb{V}}$, which has sparsity rate $\epsilon_0(n)$.

(A3) **Responsiveness.** The effectively estimated model has vanishing plug-in prediction bias, and

$$|\mathbb{E}_{\bar{\theta}}(D(Y_{V_n})) - \mathbb{E}_0(D(Y_{V_n}))| \in O(\epsilon_(n)). \qquad (3)$$

Main Result: Moving Target

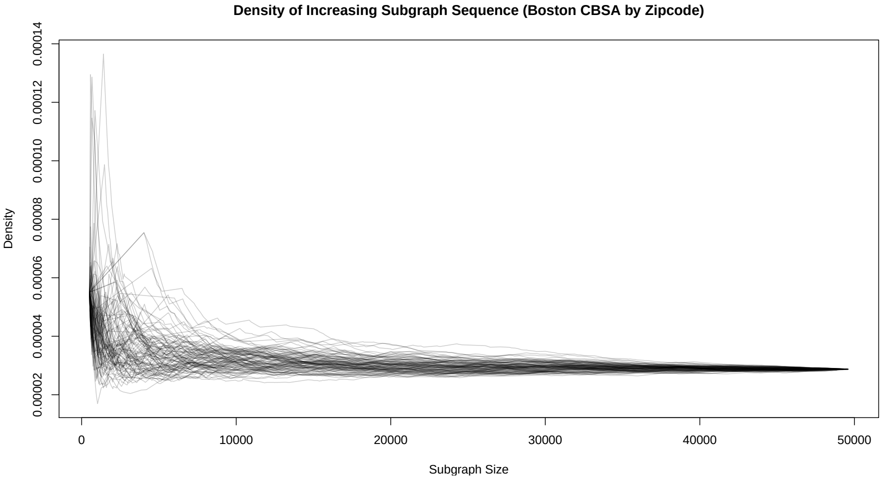
Statement

Theorem 1 (Moving target theorem).

Let (V_n) be an increasing sequence of vertex sets from $\mathbb{V}$. Suppose (A1)–(A3) hold. Then, $\bar{\theta}_{V_n}$ varies with n in the sense that for any n, there exists an $n' > n$ such that $\bar{\theta}_{V_n} \neq \bar{\theta}_{V_{n'}}$, and the MLE of the model violates Criterion 1.

Main Result: Moving Target

Intuition



Example: Poisson Regression

Setup

Question: How do firms influence collaboration dynamics?

Data: Y_V collaboration counts; X_V^{ij} indicates shared firm.

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Question: How do firms influence collaboration dynamics?

Data: Y_V collaboration counts; X_V^{ij} indicates shared firm.

Assumptions:

(E1) **Sparsity.** The true collaboration-generating process $Y_{0,\mathbb{V}}$ is sparse .

(E2) **Small firms.** All firms have finite size.

(E3) **Firms produce.** A non-vanishing fraction of firms have a positive number of expected within-firm interactions.

Example: Poisson Regression

Model and effective estimand

Model:

$$Y_V^{ij} \overset{\perp\!\!\!\perp}{\sim} \mathrm{Pois}(\exp(\theta^{(1)} + X_{V_n}^{ij}\theta^{(2)})), \tag{3}$$

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Effective Estimands:

$$\bar{\theta}_V^{(1)} = \log\left(\frac{\sum_{ij}\mathbb{E}_0(Y_V^{ij} \mid X_V^{ij} = 0)(1 - X_V^{ij})}{\sum_{ij}(1 - X_V^{ij})}\right) \tag{4}$$

$$\bar{\theta}_V^{(2)} = \log\left(\frac{\sum_{ij}\mathbb{E}_0(Y_V^{ij} \mid X_V^{ij} = 1)X_V^{ij}}{\sum_{ij}X_V^{ij}} \Bigg/ \frac{\sum_{ij}\mathbb{E}_0(Y_V^{ij} \mid X_V^{ij} = 0)(1 - X_V^{ij})}{\sum_{ij}(1 - X_V^{ij})}\right). \tag{5}$$

What's wrong with this picture?

Changing the Question

Salvaging conditional independence

What can we know about network superpopulations *without* being able to sparsity?

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Proposal: Suppose $\mathbb{P}_{0,V}$ factors into two stages. Complex **relationship** structure $R_{\mathbb{V}}$. $Y_{\mathbb{V}}$ simple conditional on relationships, i.e., for every V,

$$\mathbb{P}_{0,V}(R_V, Y_V \mid X_V) = \mathbb{P}_{0,V}(R) \prod_{i<j<n} \mathbb{P}_{0,V}(Y_V^{ij} \mid R_V, X_V). \tag{6}$$

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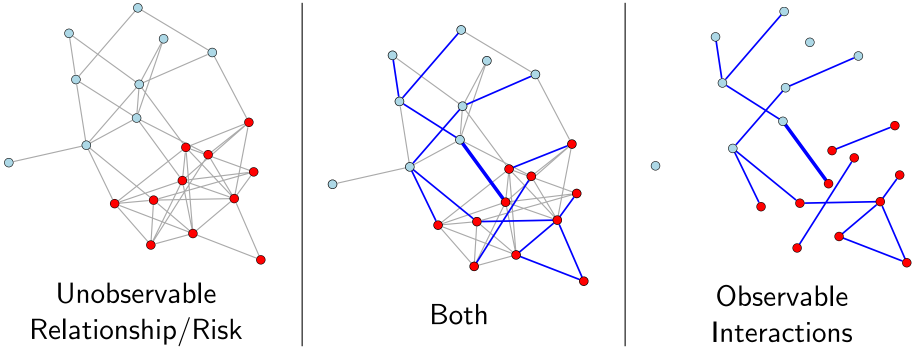
Refocus: Define sparsity-independent parameter of interest.

$$\mathbb{P}_{\theta,V}(Y_V \mid X_V) \to \mathbb{P}_{\beta,V}(Y_V \mid R_V, X_V).$$

Partial Resolution

Conditionally Independent Relationship model



Partial Resolution

Full likelihood inference?

Let $\theta \equiv (\beta, \gamma)$, with β the parameters of interest and γ the nuisance parameters.

ML estimation integrates over unobserved R_V:

$$(\bar{\beta}, \bar{\gamma}) \equiv \arg\max_{(\beta,\gamma)} \log \left[\sum_{R_V \in \mathcal{R}_V} \mathbb{P}_{\theta,V}(R_V \mid X_V) \prod_{i<j<n} \mathbb{P}_{\beta,V}(Y_V^{ij} \mid R_V^{ij}, X_V^{ij}) \right]$$

Problem solved?

Partial Resolution

Partial likelihood inference

Let $A_V = \mathbb{I}\{Y_V \neq 0\}$.

Exploit conditional distribution

$$\mathbb{P}_\beta(Y_V \mid A_V, X_V) = \frac{\mathbb{P}_\beta(Y_V \mid R_V, X_V)}{1 - \mathbb{P}_\beta(Y_V = 0 \mid R_V, X_V)},$$

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Bonus: Computation is $O(\sum_{ij} A_V)$.

Truncated Estimator

Theory

Let $Y_{\mathbb{V}}$ is a random graph process, $\mathbb{P}_{0,\mathbb{V}}$ be the true law governing this process, and $\mathcal{P}_{\Theta,\mathbb{V}}$ be a model family proposed by the investigator. Assume the following

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(T1) **CIR factorizable.** The finite-dimensional distributions of $Y_{\mathbb{V}}$ can be factorized as in Equation 6 for all sample indices V.

(T2) **Correct conditional specification.** The model family $\mathcal{P}_{\Theta,\mathbb{V}}$ correctly specifies the conditional process $\mathbb{P}_{0,\mathbb{V}}(Y_{\mathbb{V}} \mid X_{\mathbb{V}}, R_{\mathbb{V}})$, so that there exists a $\beta_0 \in B$ such that $\mathbb{P}_{\beta_0,\mathbb{V}}(Y_V^{ij} \mid X_{\mathbb{V}}, R_{\mathbb{V}}) = \mathbb{P}_{0,\mathbb{V}}(Y_V^{ij} \mid X_{\mathbb{V}}, R_{\mathbb{V}})$.

(T3) **Identification.** The model family $\mathcal{P}_{\Theta,\mathbb{V}}$ is specified so that β is identified by the truncated data $\{Y_V^{ij} : A_V^{ij} = 1\}$.

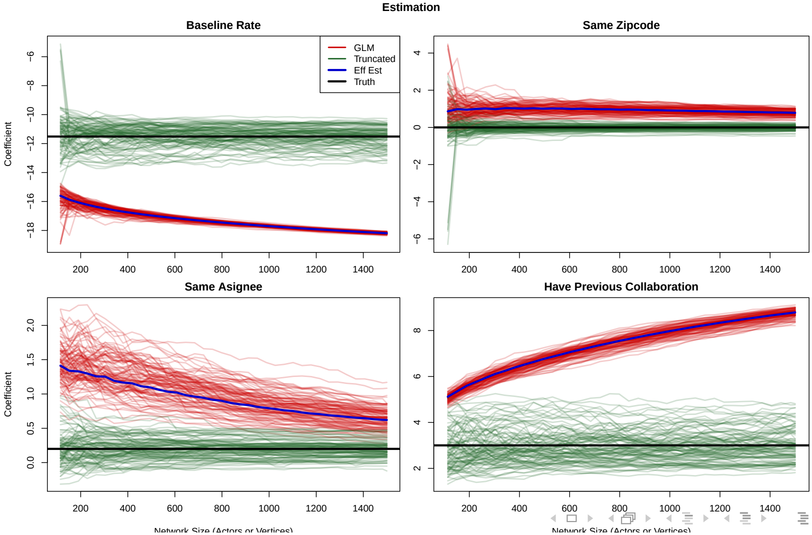
Truncated Estimator

Theorem 1 (Superpopulation Stability of Truncated Estimator).

Let $Y_{\mathbb{V}}$ is a random graph process, $\mathbb{P}_{0,\mathbb{V}}$ be the true law governing this process, and $\mathcal{P}_{\Theta,\mathbb{V}}$ be a model family proposed by the investigator. Assume that (T1)–(T3) hold.
Then the effective estimand of the MTLE does not depend on V and, in particular, $\bar{\beta}_V^{tr} = \beta_0$ for all V.

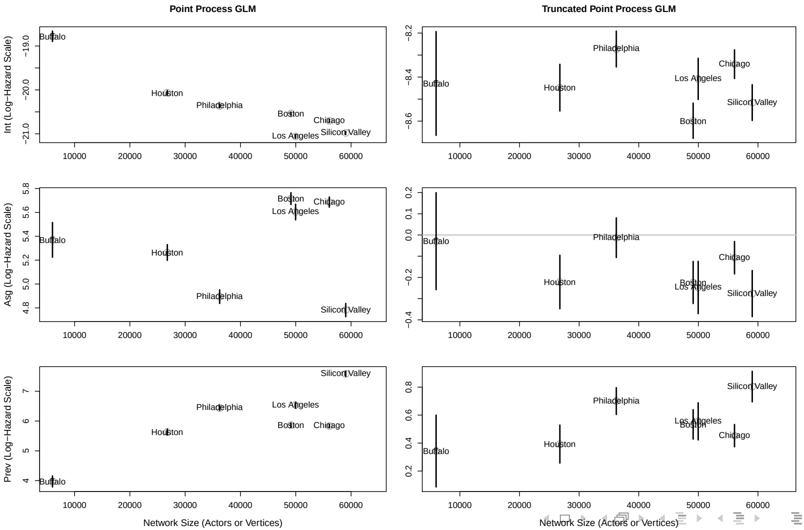
Truncated Estimator

Simulated success



Truncated Estimator

Real success



Discussion

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Modeling *less* of the system by seeking invariances make a model *more* scientifically relevant. **Invariance** can be better than a bad explanation.